Vectors and 2D Motion: Crash Course Physics #4

Crash Course: Physics
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So far, we've spent a lot of time predicting movement; where things are, where they're going, and how quickly they're gonna get there. But there's something missing, something that has a lot to do with Harry Styles. And today, we're gonna address that.

We've been talking about what happens when you do things like throw balls up in the air or drive a car down a straight road. That kind of motion is pretty simple, because there's only one axis involved. The car's accelerating either forward or backward. The ball's moving up or down. There's no messy second dimension to contend with.

But this is physics. We may simplify calculations a lot of the time, but we still want to describe the real world as best as we can. And in real life, when you need more than one direction, you turn to vectors.

(Crash Course Physics Intro)

Let's say we have a pitching machine, like you'd use for baseball practice. We're going to be using it a lot in this episode, so we might as well get familiar with how it works. We can feed the machine a bunch of baseballs and have it spit them out at any speed we want, up to 50 meters per second. the pitching height is adjustable, and we can rotate it vertically, so the ball can be launched at any angle. It also has a random setting, where the machine picks the speed, height, or angle of the ball on its own.

Suddenly we have way more options than just throwing a ball straight up in the air. And now the ball can have both horizontal and vertical qualities. At the same time.

Before, we were able to use the constant acceleration equations to describe vertical or horizontal motion, but we never used it both at once. And, we're not gonna do that today either. Instead, we're going to split the ball's motion into two parts, we'll talk about what's happening horizontally and vertically, but completely separately. And we'll do that with the help of vectors.

Vectors are kind of like ordinary numbers, which are also known as scalars, because they have a magnitude, which tells you how big they are. But vectors have another characteristic too:
direction. Previously, we might have said that a ball's velocity was 5 meters per second, and, assuming we'd picked downward to be the positive direction, we'd know that the ball was falling down, since its velocity was positive.

In other words, we were taking direction into account, it we could only describe that direction using a positive or negative. So we were limited to two directions along one axis. But vectors change all that. Now, instead of just two directions we can talk about any direction.

It might help to think of a vector like an arrow on a treasure map. You could draw an arrow that represents 5 kilometers on the map, and that length would be the vector's magnitude. But you need to point it in a particular direction to tell people where to find the treasure. Which is actually pretty much how physicists graph vectors. You take your two usual axes, aim in the vector's direction, and then draw an arrow, as long as its magnitude.

Like say your pitching machine launches a ball at a 30 degree angle from the horizontal, with a starting velocity of 5 meters per second. We can just draw that as a vector with a magnitude of 5 and a direction of 30 degrees. Let's say your catcher didn't catch the ball properly and dropped it. Then just before it hits the ground, its velocity might've had a magnitude of 3 meters per second and a direction of 270 degrees, which we can draw like this.

That's why vectors are so useful, you can describe any direction you want. But there's a problem, one you might have already noticed. You can't just add or multiply these vectors the same way you would ordinary numbers, because they aren't ordinary numbers. To do that, we have to describe vectors differently.

When you draw a vector, it's a lot like the hypotenuse of a right triangle. The vector's magnitude tells you the length of that hypotenuse, and you can use its angle to draw the rest of the triangle. Right angle triangles are cool like that, you only need to know a couple things about one, like the length of a side and the degrees in an angle, to draw the rest of it. It's all trigonometry, connecting sides and angles through sines and cosines.

Which is why you can also describe a vector just by writing the lengths of those two other sides. In fact, those sides are so good at describing a vector that physicists call them components.

So let's get back to our pitching machine example for a minute. We said that the vector for the ball's starting velocity had a magnitude of 5 and a direction of 30 degrees above the horizontal. We can draw that out like this. That's all we need to do the trig. The length of that horizontal side, or component, must be 5cos30, which is 4.33. The same math works for the vertical side, just with sine instead of the cosine. So we know that the length of the vertical side is just 5sin30, which works out to be 2.5. So our vector has a horizontal component of 4.33 and a vertical component of 2.5.

In what's known as unit vector notation, we'd describe this vector as v = 4.33i + 2.5j. The arrow on top of the v tells you it's a vector, and the little hats on top of the i and j, tell you that they're the unit vectors, and they denote the direction for each vector. i just means it's the direction of what we'd normally call the x axis, and j is the y axis. (4:51) You'll sometimes another one, k, which represents the z axis.

And, if you want to add or subtract two vectors, that's easy enough. We just separate them each into their component parts, and add or subtract each component separately. So 2i plus 5j added to 5i plus 6j would just be 7i plus 9j. And -2i plus 3j added to 5i minus 6j would be 3i minus 3j. Multiplying by a scalar isn't a big deal either. You just multiply the number by each component. So 2i plus 3j times 3 would be 6i plus 9j.

The unit vector notation itself actually takes advantage of this kind of multiplication. I, j, and k are all called unit vectors because they're vectors that are exactly one unit long, each pointing in the direction of a different axis. So when you write 2i, for example, you're just saying, take the unit vector i and make it twice as long. But that's not the same as multiplying a vector by another vector. That's a topic for another episode.

So now we know that a vector has two parts: a magnitude and a direction, and that it often helps to describe it in terms of its components. And when you separate a vector into its components, they really are completely separate. In other words, changing a horizontal vector won't affect it's vertical component and vice versa. And we can test this idea pretty easily.

Let's say you have two baseballs and you let go of them at the same time from the same height, but you toss Ball A in such a way that it ends up with some starting vertical velocity. With Ball B, it's just dropped. In this case, Ball A will hit the ground first because you gave it a head start. Now, what happens if you repeat the experiment, but this time you give Ball A some horizontal velocity and just drop Ball B straight down? Which ball hits the ground first? It's kind of a trick question because they actually land at the same time. It doesn't matter how much starting horizontal velocity you give Ball A- it doesn't reach the ground any more quickly because its horizontal motion vector has nothing to do with its vertical

motion.

With this in mind, let's go back to our pitching machines, which we'll set up so it's pitching balls horizontally, exactly a meter above the ground. Then we get out of the way and launch a ball, assuming that up and right each are positive. How do we figure out how long it takes to hit the ground? That's easy enough- we just completely ignore the horizontal component and use the kinetic equations the same way we've been using them.

In this case, the one we want is what we've been calling the displacement curve equation -- it's this one. We just add y subscripts to velocity and acceleration, since we're specifically talking about those qualities in the vertical direction. Now we can start plugging in the numbers. The ball's displacement, on the left side of the equation, is just -1 meter. There's no starting VERTICAL velocity, since the machine is pointing sideways. And the vertical acceleration is just the force of gravity. Now all we have to do is solve for time, t, and we learn that the ball took 0.452 seconds to hit the ground. Its horizontal motion didn't affect its vertical motion in any way. But sometimes things get a little more complicated -- like, what about those pitches we were launching with a starting velocity of 5 meters per second, but at an angle of 30 degrees? Well, we can still talk about the ball's vertical and horizontal motion separately. We just have to separate that velocity vector into its components. Just like we did earlier, we can use trigonometry to get a starting horizontal velocity of 4.33 m/s and a starting vertical velocity of 2.5 m/s. Now we're equipped to answer all kinds of questions about the ball's horizontal or vertical motion. Here's one: how long did it take for the ball to reach its highest point? We already know SOMETHING important about this mysterious maximum: at that final point, the ball's vertical velocity had to be zero. That's because of something we've talked about before: when you reverse directions, your velocity has to hit zero, at least for that one moment, before you head back the other way. So, in this case, we know that the ball's starting vertical velocity was 2.5 m/s. And we know that its final vertical velocity, at that high point, was 0 m/s. Finally, we know that its vertical acceleration came from the force of gravity -- so it was -9.81 m/s^2, since up is Positive and we're looking for time, t. Fortunately, you know that there's a kinematic equation that fits this scenario perfectly -- the definition of acceleration. By plugging in these numbers, we find that it took the ball 0.255 seconds to hit that maximum height. So, describing motion in more than one dimension isn't really all that different, or complicated. You just have to use the power of triangles. In this episode, you learned about vectors, how to resolve them into components, and how to add and subtract those components. We also talked about how to use the kinematic equations, to describe motion in each dimension separately. Crash Course Physics is produced in association with PBS Digital Studios. You can head over to their channel to check out amazing shows like The Art Assignment, The Chatterbox, and Blank on Blank. This episode of Crash Course was filmed in the Doctor Cheryl C. Kinney Crash Course Studio, with the help of these amazing people and our Graphics Team is Thought Cafe.