Rotational Motion: Crash Course Physics #11

Crash Course: Physics
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At this point, we've covered a lot of the basics when it comes to how things move, but we've mostly been focusing on only one type of motion: translational motion, which is when something moves through space, but doesn't rotate. But rotational motion is also a thing, and an important one.

For example, the spin of a football, both the soccer and the non-soccer kind, will affect the way it flies through the air. But the physics of rotational motion isn't all that different from the physics of translational motion. It still involves things like position, velocity, and acceleration, and many of the equations we used to describe rotational motion will look really familiar to you, but there are some important differences, like instead of positions, there are angles. Instead of points along a line, you follow points along an arc, and there are times when rotational motion can explain things that sound impossible, but are actually true. Like when a point on a spinning wheel is actually standing still. So the rules here are the same, but rotational motion has what you might call its own circular logic.

(Intro)

When it comes to translational motion, we tend to talk about position in terms of x and y. Where is this object horizontally, and where is it vertically? Those axes make sense, because we're usually tracking the object's motion along those directions. But for rotational motion, we really want to know the object's angle: what we call theta.

Say we have a big disc with a dot painted on it. If we call the right side of the disc our starting point, then when the dot is at the right, its angle will be zero, and when the dot is at the left, its angle will be 180 degrees, half of a full circle. But even though we've been measuring angles using degrees until now, there's another unit that physicists use a lot. This will be the primary unit we use in this episode and the next. That unit is called the radian, and its name comes from the fact that it's based on the radius of the circle.

If you think back to basic geometry, you'll recall that the circumference of a circle is just 2 times pi times the circle's radius. Radians describe angles by essentially telling you how much of that circumference is covered by a given angle.

So, 360 degrees, which is a full circle's worth of angles, would be 2 pi radians. 180 degrees, or half a circle's worth of angles, would be pi radians, and to convent any number of degrees to radians, you just multiply the degrees times pi and divide by 180.

So now we know how to describe the angle of something that's rotating, but what about the velocity of its rotation? Well, we've already learned that plain old translational or linear velocity is a measure of an object's change in position, and in the same way, rotational velocity is a measure of an object's change in angle. This is known as angular velocity, and it's represented by the lowercase Greek letter, Omega, which I want to point out looks a bit like a 'w', but isn't a 'w'. And as you might have guessed by now, angular velocity is the derivative of the rate of change of angular displacement with respect to time.

But we can also describe an object's rotation in terms of its tangential velocity. This is the same type of velocity we used when we talked about the physics of uniform circular motion. Remember? With the key spinning around on a string on that vomit causing carnival ride? In those cases, we described how, when an object moves along a circular path, its velocity is perpendicular to the radius of the circle in the direction of the motion. When you think about it, any rigid object rotating around a fixed axis is basically a set of points all moving around along circular paths.

So, at any given moment, each of those points will have a tangential velocity that depends on the path it moves through, specifically the radius of that path. In fact, its tangential velocity will be equal to its angular velocity times the radius. It's easy to see why this makes sense, if you picture the spokes of a rotating wheel. All of the points along each spoke have to have the same angular velocity, because they all cover the same angular distance in the same amount of time. But to get from, say, the right hand side of the circle to the bottom, the points on the outside of the wheel will pass through a much bigger arc, covering more distance basically, than the points on the inside, so the further that a point on the spoke is from the center of the wheel, the greater its tangential velocity has to be.

Like circular motion, rotational motion can also be periodic, periodic being when the rotation repeats after a set amount of time, which is represented by capital 'T', also called the period. And the equations that describe periodic motion are pretty much the same as the ones we used for a single point moving along a circular path. So the frequency, or number of rotations that happen every second, is equal to one, divided by the period. But frequency and angular velocity are really just two different ways of describing the same thing -- they just use different units. Frequency is measured in rotations -- or revolutions -- per second, and angular velocity is measured in radians per second. And one revolution is equal to the circumference of the circle: 2 pi radians.

So, in order to convert from frequency to angular velocity, all you need to do is multiply the frequency by 2 pi. Now, there’s a special case when it comes to the velocity of rotating objects, and that’s what’s known as rolling without slipping. This kind of motion shows up in real life all the time. It’s what happens to your car’s tires when you drive down the street, as long as you aren’t skidding -- which, let’s hope you aren’t. And it’s what a train’s wheels do as they move along the track. But it turns out that the translational velocity at the bottom of the wheel is super weird. Mainly, because at any given moment, the point at the bottom of the wheel doesn’t have a translational velocity. In other words, it doesn’t actually move.

To figure out why, let’s experiment with a bicycle wheel. If you roll the wheel along the floor for one full rotation, that means that the entire circumference of the wheel will touch the floor, one point at a time. And the center of the wheel will move forward by a distance that’s equal to the circumference of the wheel -- aka its radius, times 2 pi. And the time it took to move that distance was equal to the period of the motion. So, the translational velocity of the center was equal to the radius, times the angular velocity.

Now, what about the top of the wheel? It has the same translational velocity as the center of the wheel, plus the tangential velocity that comes from the wheel’s rotation. Because, at the top of the wheel, the tangential velocity is pointing in the direction the wheel is rolling in. And in the same way, the bottom of the wheel has the same translational velocity as the center of the wheel, minus the tangential velocity that comes from the wheel’s rotation. Because at the bottom of the wheel, the tangential velocity is pointing opposite to the direction the wheel is rolling in.

Here’s the weird part: We just saw that the translational velocity of the wheel is equal to the radius, times the angular velocity. And we know that in general, the magnitude of tangential velocity is also equal to the radius, times the angular velocity. So, the top of the wheel will be moving exactly twice as fast as the center of the wheel, relative to the ground. Because, to get its total velocity, you add the translational velocity to the tangential velocity. But the bottom of the wheel won’t be moving at all... because its total

velocity is its translational velocity minus the tangential, since they're moving in opposite directions.

As a result, the total velocity at the bottom of the wheel is zero. Even though the wheel is clearly moving relative to the ground. But if you look at the wheel's motion at any given instant, you'll see that whatever point is at the bottom of the wheel can't be moving relative to the ground. If it was moving relative to the ground... that would be what we call slipping. Like when a car is skidding on an icy ground: The wheel isn't turning, but the bottom of the wheel is moving in relation to the ground, because it's sliding along on top of it. But this wheel isn't slipping -- its bottom has a total velocity of zero, because its velocities cancel out.

OK: I know I just blew your mind, so while you put your head back together, I want to talk about one more basic quality of rotational motion: angular acceleration. Based on what you already know about acceleration, you can already guess that angular acceleration is the derivative of angular velocity. It's represented by the lowercase Greek letter alpha, and it describes how an object's angular velocity is changing over time. And as an object rotates, each point on it can actually accelerate in two different ways.

Radial acceleration is another term for what we've been calling centripetal acceleration up until now. It's the acceleration inward of any point on our rotating object, and it's equal to the angular velocity, squared, times the radius. But there's also tangential acceleration, which describes whether an individual point on a rotating object is speeding up or slowing down. So, like linear velocity, tangential acceleration depends on the distance between the point and the center of the object. More specifically, it's equal to the angular acceleration, times the radius.

So you see, angular position, velocity, and acceleration relate to each other in much the same ways that linear position, velocity, and acceleration do. This allows us to talk about rotational motion with terms and equations that are familiar to us, once we've gotten the basics of translational motion under our belts.

Next time, we'll see how how the logic of rotational motion applies to another idea: momentum! For now, you learned about the qualities of rotational motion, including angular position, angular velocity, periodic motion, and the special case of rolling without slipping. We also talked about angular acceleration, as well as constant angular acceleration.

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