Derivatives: Crash Course Physics #2

Crash Course: Physics
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=====Introduction(00:00)=====

Every discipline of science has its very own special language, the way it communicates the ideas that it investigates. For example, biology finds order in the world, by giving every living thing a name in Latin. Chemistry has a system of prefixes, suffixes, and numerals to tell you, in a word or two, the exact composition of an atom, or a compound.

Physics has to communicate its ideas differently. The language of physics is mathematics. Because if you're trying to describe how the worlds works, you really have to know how things relate to each other in a mathematical way. For example, we've been talking a lot about position, velocity, and acceleration, and how they're all connected. Velocity is a measure of your change in position, and acceleration is a measure of your change in velocity. They're connected, one quality will describe how the other is changing. And the way we describe change in mathematics is through calculus.

Calculus explains how and why things change, using derivatives, which help you determine how an equation is changing, as well as with integrals, which you can use to calculate the area under a curve. Derivatives and integrals themselves are closely connected. But let's start with derivatives.

You probably won't be able to go straight from this lesson to your calculus final. But hopefully, in about 10 minutes, you will be able to understand some of the maths that scientists have been using to think about physics, for at least 400 years or so. And you'll also have a new way to fight speeding tickets. You know, just in case.

(Crash Course Physics Intro)

===== Speeding Ticket (01:31)=====

Last time, we talked about that unfortunate incident where you got a speeding ticket. Your speedometer was broken, but because we knew your acceleration, we were able to calculate how fast you were going when the cops pulled you over. So now, let's talk about what happens next. Say the police drive off. You're ready to get back on the road, so you hit the gas and zoom forward, moving faster and faster. But in this scenario, we don't know your acceleration; we only know how much your position is changing over time.

In this instance, your position happens to be equal to the amount of time you've been driving, squared. So we'd write that as the equation $x=t^2$. 20 seconds in, you pass a detector with a sign that tells you your speed. You keep driving, foot still on the gas, before you realize what number you saw on the sign. And, oh no! You just a speeding ticket in the last episode, for doing a 126 km/h in a 100 km/h zone, and now the detector says you're going even faster!

Now you want to know if the number on the detector is accurate, in other words, you want to find your velocity, at the exact moment you passed it. That velocity is just a measure of your change in position, it's derivative. So to find your velocity, we'll need to find the derivative of your position. And in order to determine that, we first need to talk about limits.

===== Limits (2:43) =====

Not speed limits, I mean the derivatives kind. I'll explain...

Limits are based on the idea that if you have an equation on a graph, you can often predict what it's going to look like at one point by just knowing what it looks like at the surrounding points. For example, let's say you have a graph of $x=t^2$, from the speeding scenario above. And you want to find out how your position is changing at the exact moment that time is equal to zero. That is what we'd call the limit as t approaches zero.

So you take a look around what's happening around t=0. At t=1, x is 1. At t=0.5, x is 0.25. And at t=0.1, x is 0.01. You can probably tell that as we get closer and closer to t=0, your value of x is getting closer to zero, too. That's what mathematicians mean when they talk about a limit. Limits are useful because they can help predict what happens as you make intervals smaller. An interval is just a range on a graph. It's the space between two point on the horizontal axis.

So the first thing we can try is calculating your average velocity over the interval from 15 to 20 seconds. To do that, we use and equation we talked about last time, your average velocity, which is equal to the change in your position divided by the change in time. That turns out to be 35 m/s. Problem is, it's still just an average, it's not exactly how fast you were going after 20 seconds of acceleration, when you passed the detector.

Because of limits, we know that you could get a little closer to the right number by calculating your average over smaller and smaller intervals. Then you'd see that the number seemed to be getting closer and closer to 40 m/s. Which means that you're going to need to slow way down if you don't want to get your second speeding ticket of the day.

But that's the idea of derivatives, you can use infinitely tiny intervals to figure out exactly how an equation is changing at any moment. You can even come up with an equation to describe the change. That's exactly what velocity is, an equation that describes change in position. And acceleration describes change in velocity. So we'd call velocity the derivative of position, and acceleration the derivative of velocity.

===== Power Rule(4:45) =====

Now, when it comes to how you express a derivative in writing, mathematicians have come up with shortcuts. Like what's known as the Power Rule. As the name suggests, it's used for equations with variables raised to powers, or exponents, as long as the exponent is a number. For example, $x=t^2$ would work with the power rule, because t is raised to the power of 2. The power rule says that for these kinds of equations, to calculate the derivative, all you need is one weird trick. Take the number of that exponent, in this case 2, and stick it in front of the variable. Then you subtract 1 from the exponent. And that's your derivative.

So the derivative of $x=t^2$ is just 2t. Which means that no matter how many seconds you've got your foot on the gas, your velocity will be 2t, so double the number of seconds. After 5 seconds, you were going a modest 10m/s. But after 20 seconds, you were going a full 40 m/s. Which is not good. We'd write that like this: Where dx/dt (dx over dt) is just a way of saying that we're taking the derivative of the part of the equation that involves t. Or, as a mathematician would put it, we're taking the derivative of x with respect to t. You'll also sometimes see this written in a different way. If f of t [f(t)] is equal to t squared (t^2), then f prime of t [f'(t)] is equal to 2t.

===== Power Rule Examples (5:58) =====

Now let's try to find a couple more derivatives using the power rule: $x=7t^2$ is another power-style equation. It has a variable, t, raised to a power, 6, with a number in front of it ,7. The first thing we do is take the exponent, and stick it in front of the variable. But there's already a number in front of t; 7. So we end up multiplying them, 7 times 6 is 42. Then we subtract 1 from the power that t is raised to. So we end up with $42t^5$.

Same goes for equations where the exponents are fractions or decimals. So the derivative of $t^{½}$ is the half t to the negative on half. It works for negative exponents too, the derivative of t^{-2} is just -2t to the negative third.

===== Trigonometric Derivatives (6:42) =====
Now there are a few more equations whose derivatives you should understand. Trigonometry, which we use to calculate the angles and sides of triangles, is going to come up a lot in physics, because we'll be using right angle triangles all the time. So it's a good idea to know how to find the derivatives of sin(x) and cos(x). Sine tells you that if you have a right angle triangle, and x is an angle in that triangle, then sin(x) will be the length of the side opposite that angle divided by the hypotenuse. Cosine does the same thing, just with the side next to that angle divided by the hypotenuse. So their graphs tell you what those ratios will be, depending on the angle.

We can actually try to guess the derivative of sin(x) just by looking at its graph. You can see that the curve has turning points every so often, at x=-90°, x=90°, and so on, repeating every 180°. Meaning, at those points, the equations aren't changing at all, so the derivative at these turning points is also going to be exactly zero. Let's pull up another graph where we'll plot the derivative, and put little dots where we'll know it'll be zero.

Now, what's happening between those turning points? Well, from -270° to -90°, sin(x) is decreasing. In other words, its change, and therefore its derivative, must be negative. Then, from -90° to 90°, sin(x) is increasing, so it'll have a positive derivative. And so on... There are actually a lot more clues in this graph to help us find the derivative, but we already know enough to make a decent guess.

If we smoothly connect the dots on the graph of our derivative, keeping in mind where the curve should be positive and where it should be negative, hey, this derivative is looking a whole lot like the graph of cos(x). That's because it is. The derivative of sine is just cosine, and that is gonna come up a lot. And so will these, which you can work out on your own by repeating what we just did with the graphs of sin(x) and cos(x).

===== Derivative of e^x (8:30) =====
Another important derivative that comes up a lot is a very special case, and that's e^x. the derivative of e^x is just e^x. Yep, that's it. No matter what. In fact, that's one way to define e, which is kind of like pi in the sense that it's a simple letter representing a very specific, irrational number, about 2.718, but with more digits after the decimal point that go on forever. It has all sorts of uses in calculus, but it also shows up when you're studying things like finance and probability.

===== Summary and Conclusion (9:00) =====
Armed with all these ways to find derivatives you could take pretty much any equation of your position and calculate its derivative, and therefore your velocity. In the same way, you could take the derivative of your velocity and find your acceleration. But there's a whole other part of calculus that we haven't talked about yet, integrals, which will let you do this backwards. With integrals, you can use your acceleration to find your velocity, and your velocity to find your position. But we'll save that for next time.

Today, you learned about limits, and that derivatives user them to describe how an equation is changing. We also talked about a few different kinds of derivatives; powers, constants, trigonometry and e^x.

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