Simple Harmonic Motion: Crash Course Physics #16

Crash Course: Physics
https://youtube.com/watch?v=jxstE6A_CYQ
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In June 2001, officials in London unveiled a striking new feat of engineering - the millennium bridge - a pedestrian bridge spanning the river Thames. It promised to be very useful and it was cool to look at but it was closed almost immediately, because when people used the bridge it swayed back and forth noticeably, due to the force of their footsteps.

Undeterred, people kept using the bridge, but as they walked they began to lean into the swaying, to keep themselves from falling over. And that only made things worse. Eventually the motion of the bridge became so severe that the bridge took on the shape of a giant S.

Essentially, a horizontal wave. the bridge had to be closed and the engineers took nearly two years to fix the problems. So what was wrong with the millennium bridge? And why didn't the engineers foresee the problem? The answer lies in oscillations.

Theme music.

The physics that caused the swaying of the millennium bridge has to do with oscillations, or back and forth motion. Or more specifically, it has to do with simple harmonic motion, where oscillations follow a particular, consistent pattern.

But before we had the millennium bridge as a real life example, physicists often described simple harmonic motion in terms of a ball attached to a horizontal spring, lying on a table.

While it's lying there at rest, it's in equilibrium, and when you move the ball so that it stretches the spring then let go, the ball keeps moving back and forth forever... in a friction-less world.

That back and forth motion caused by the force of the spring is simple harmonic motion.

Now, we want to know two things about this oscillating ball - what kings of energy does it have, and what's it's maximum velocity?

To better understand what's happening to the ball, let's start with it's energy. As the ball compresses and stretches the string, both kinetic energy and potential energy come into play.

Kinetic energy is the energy of motion and as the ball moves there are two points - the turning points where it's not moving. One point is where the spring is compressed all the way and the other is where it's stretched all the way, and the distance between these two points and the equilibrium point is called the amplitude.

At those two turning points, the ball won't have any kinetic energy, since it isn't moving. Instead, all of the balls energy will be potential energy from the spring - half of the spring constant plus the amplitude squared.

Now, as the ball moves towards the middle, its kinetic energy starts to increase because it's moving faster and faster. And at the same time, its potential energy decreases, keeping its total energy the same.

And exactly in the middle of the balls motion, at the equilibrium point, its potential energy goes down to zero. The ball is back where it started the the ball is no longer pulling on it.

Its kinetic energy, on the other hand, has reached its maximum, which means that at the point the total energy of the ball will be equal to half of its mass times its maximum velocity squared.

Now we have two equations for the total energy in this oscillating spring, which we can combine into one equation, and if we use algebra to move around its variables we can start to answer the second question we had about the ball.

We wanted to know the balls maximum velocity, and this equation tells us that it's equal to the amplitude times the square root of the spring constant divided by its mass.

So we've answered our two questions about the ball on the spring, we know about its energy and we have an equation for its maximum velocity.

But there's a lot more going on with this ball than just its energy and velocity. It also has properties like period, a frequency, and an angular velocity plus its position changes with time.

You might recognized those terms because we already talked about them in our episode on uniform circular motion, and that's no coincidence. Simple harmonic motion is actually a lot like uniform circular motion - mathematically speaking.

You can see this for yourself if you compare that ball's motion on the spring to an object in uniform circular motion, say a marble, moving along a ring at a constant speed.

Ok, I admit, it might seem like kind of a weird comparison at first. For one thing the ball on the spring is moving in one dimension, while a marble moving along a circular path is in two dimensions.

But what if you take the ring and look at it from the side. The marble keeps moving along its circular path, but to you it looks like its just moving back and forth along a straight line.

Not only that, but it looks like this marble is stopping momentarily as it changes direction and moving faster as it gets close to the middle, which is exactly the same way the ball was moving on the spring.

now let's take this comparison one step further. Let's assume the radius of the ring is the same as the amplitude of the balls motion on the spring, and that the marble's constant speed along the ring is equal to the maximum speed of the ball on the spring.

In that case, if you did the maths, you'd find that the equation for the marble's velocity - when you look at it edge on - is exactly the same equation that described the velocity of the ball on the spring.

So let's recall what we know about uniform circular motion, to see what it can tell us about simple harmonic motion.

We know that it takes for the marble to move around the ring once is called the period. We also know that the period will be equal to the circumference of the ring divided by the marble's speed, and the radius of the circle is the same as the balls amplitude on the spring, so its circumference will be equal to two times pi times the amplitude.

This means that the period will be equal to two times pi times the amplitude divided by the marble's speed - which again is the same as the ball's maximum speed as it moves on the spring.

and we can simplify that equation since we know that the maximum speed of the ball is equal to the amplitude times the square root of the spring constant divided by the mass.

So, the period of the marble's motion around the ring is equal to two pi times the square root of m over k.

Now, we've also talked about the frequency of uniform circular motion. It's the number of revolutions the marble makes around the ring every second, and its equal to one divided by the period.

In this case, the frequency will also be equal to one over two pi times the square root of k over m. That'll apply to the ball on the spring too, because the rules are the same.

Finally, there's angular velocity to consider. In uniform circular motion, we've described it as the number of radians per second that the marble covers as it moves around the ring. And angular velocity is jut equal to the frequency times two pi, which means that in the case of the ball on the spring is equal to the square root of k over m.

So now, with the help of our knowledge about circular motion, we can understand the period, frequency and angular velocity of the ball's simple harmonic motion as it oscillates on the spring.

But there's one more question - how does the balls position change over time? To find out, we'll have to analyze the marble's motion along the ring again, and the answer will involve some trigonometry, but it's not particularly complicated trig, so you'll be fine.

At any given point along its path, the marble will be at a certain angle to the right hand side of the ring, and the cosine of that angle will be equal to its horizontal distance from the centre of the ring divided by the rings radius. We already know that the radius of the ring is the same as the amplitude of the balls motion along the spring.

And if you turn the ring so it looks like a line again, you can see that the marble's horizontal distance from the centre of the ring is the same as the balls distance from the equilibrium points.

So the cosine of theta is equal to the balls position, divided by its amplitude. In other words the balls position is equal to the amplitude plus the cosine of the angle. And we can simplify this equation too.

In the same way that distance is equal to velocity multiplied by time, the angle is equal to the angular velocity multiplied by time.

So we can write the equation for the position of the ball as x equals A cosine omega t. And when you graph the equation, something interesting happens - it looks like a wave.

We'll be talking a lot more about waves in our next three episodes, but for now it's helpful just to see the connection here: for an object in simple harmonic motion, the graph of its position versus time is a wave. Which is why the swaying of the millennium bridge looked like a wave.

Speaking of the bridge, we can now better understand what happened to it. The bridges shimmy was the result of oscillation, but it was made worse by another culprit: resonance.

Resonance can increase the amplitude of an oscillation by applying force at just the right frequency, kind of like how you can get a kid on a swing set to swing higher by pushing at just the right moment.

The engineers of the millennium bridge were reminded of that, the hard way. When pedestrians on the bridge began to lean into its swaying, they created resonance - they amplified the amplitude of the oscillation.

And the engineers of the bridge did account for the oscillations caused by resonance when they designed it. But they only considered the vertical oscillations, the kind that would have made the bridge bounce up and down.

They didn't realize that they'd also had to factor in the horizontal swaying caused by people walking. So it was only a tiny bit of swaying a first but it got a lot worse because people were leaning into their steps causing resonance.

In the end, engineers had to apply a series of changes to the bridge to counteract its oscillations, because if there's one thing you don't want your bridge to be doing, it's the wave.

Today you learned about simple harmonic motion, the energy of that motion, and how we can use math of uniform circular motion to find the period, frequency, and angular velocity of a mass on a spring. We also described how the position of an object in simple harmonic motion changes over time.

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